Problem Set #3 1: Z-scores and Probability

1. Assume heights are normally distributed with a mean of 68 inches and a standard deviation of 4 inches.
   1. Use z-scores to determine the probability that an Amherst student is at least 5'3".  Hint: consult the unit normal table, which can be downloaded from Moodle.

z = x - mean / sd  
 = 63 – 68 / 4  
 = -5 / 4  
 = -1.25  
According to the Z-table, .8944 of the standard normal curve lies above –1.25 (Body). This suggests that the proportion of the Amherst student body that is over 5'3" is .8944 or 89%.

* 1. Use z-scores to determine the proportion of people that are between 5'4" and 6'0" (i.e., at least 5.4", but not taller than 6'0").  Use the same procedure as in #1.

x - mean / s ≤ z ≤ x - mean / s

64 – 68/ 4 ≤ z ≤ 72 – 68 / 4

-4/ 4 ≤ z ≤ -4 / 4

-1 ≤ z ≤ -1

According to the table, .3413 of the standard normal curve lives between –1 and the mean (0) and .3413 of the standard normal curve lives between the mean (0) and +1. Therefore, the area between these two points is .3413 \*2 = .6826. This suggests that the proportion of AC students that fall between 5'4" and 6' is .6826 or 68%.

Alternatively, you could have calculated the area in the tail below -1 (.1587) and the area in the body beneath +1 (.8413); .8413 -.1587 = .6826.

* 1. If I select a person at random, what would we estimate is the probability they will be over 6'0"?

The proportion in the tail above +1 is .1587, therefore the probability someone will be over 6’0” is .1587

1. A physical fitness association is including the mile run in its secondary school fitness test. The time for this event is approximately normally distributed with a mean of 450 seconds and a SD of 40 seconds. If the association wants to designate the fastest 10% as “excellent” what time should the association set as their cutoff?

According to the unit normal z-table, a z score of -1.28 will have approximately 10% of scores in the tail (.1003). Therefore, we have to find the X score that corresponds to a z score of -1.28. Remember we are interested in the fastest times, meaning the lowest scores.

X = 450 + -1.28 (40) = 398.8 seconds