Problem Set #4: CLT and Confidence Intervals

1. Fill in the blanks: According to the central limit theorem, the *sampling* distribution of a population will be approximately *normal* if n is sufficiently large (n > *30*). Also, the population parameter should be equal to the mean of the *sampling distribution* and the *standard error* will be equal to σ/√𝑁
2. What is the difference between:
	1. A sampling distribution
	2. A population distribution
	3. A sample distribution

A **sampling distribution** is a distribution of **means of samples** that are randomly selected from a population. A sampling distribution is represented with a **smooth curve**.

A **population distribution** is a distribution of **scores from a population.** This is a type of frequency distribution**.** A population distribution is represented by a **smooth curve.**

**A sample distribution** is a distribution of **scores from a sample**. This is also a type of frequency distribution. A sample distribution is represented by a **histogram/bar graph**.

1. A researcher examines the relationship between delinquent behaviors and poor verbal abilities in teenagers. They administer a verbal IQ test to a sample of 81 incarcerated juvenile delinquents, you find that the sample mean verbal IQ is 103. The verbal IQ test is known to have a *μ* = 107 and a *σ =* 15 in the general population of teenagers.
	1. Assuming that the population mean and SD for juvenile delinquents is the same as that for the general population of teenagers, what is the probability of selecting a sample with a mean of 103 or lower?

𝜇𝑀=107

𝜎𝑀= σ𝑥/√𝑁 = 15/√81 = 15/9=1.67

𝑍=(𝑀−𝜇𝑀)/𝜎𝑀 = (103−107)/1.67 = -2.40

We can’t find the exact probability of a mean, so the best we can do is find the proportion at or lower that this value in the z distribution. Proportion in lower tail= .0082; so the probability of obtaining a simple mean as low or lower than 102 is .0082.

* 1. Do you think that juvenile delinquents have the same population mean and SD for verbal IQ as the general population of teenagers? Explain your answer.

No, this is highly unlikely because it’s highly unlikely to draw a random sample with a mean as low as 103 from the general population of teenagers. Juvenile delinquents are likely to actually have a lower population mean and SD for verbal IQ- they likely represent a separate population.

1. In a previous problem set, I told you that these were the number of hours of sleep that I had received each night for the past week: 6, 7, 7, 6, 7, 4.5, 8. Last time I asked you to find the mean, median, mode, variance and standard deviation for these data.
	1. Now I want you to find the 90% and 95% CI for the average number of hours I sleep per night based on the numbers I obtained for that week.

Remember this is a sample not a population

Mean = 6.5

 s = 1.12

 SE = 1.12 / (sqrt(7)) = .42

 df = 7-1 = 6

Plug these values into the formula for the CI

90% CI = Mean ± tα/2 \* (s / sqrt(n))

 = 6.5 ± 1.943 \* (.42)

 = 6.5 ± .72

 90% CI = **[5.68 – 7.32]**

95% CI = Mean ± tα/2 \* (s / sqrt(n))

 = 6.5 ± 2.447 \* (.42)

 = 6.5 ± 1.03

 95% CI = **[5.47 – 7.53]**

1. Provide an interpretation of what each CI tells us

We are 90% certain that the average number of hours a night I sleep (or the population mean) is between 5.68 and 7.32 hours.

We are 95% certain that the average number of hours a night I sleep is between 5.47 hours and 7.53 hours.